Exercise 47

Show that $\frac{d}{dx}\arctan(\tanh x) = \operatorname{sech} 2x$.

Solution

Differentiate the given function by using the chain rule. Note the result of Exercise 16: $\cosh 2x = \cosh^2 x + \sinh^2 x$.

$$\begin{aligned}
\frac{d}{dx}\arctan(\tanh x) &= \frac{1}{1+(\tanh x)^2}\cdot\frac{d}{dx}(\tanh x) \\
&= \frac{1}{1+\tanh^2 x}\cdot(\operatorname{sech}^2 x) \\
&= \frac{1}{1+\frac{\sinh^2 x}{\cosh^2 x}}\cdot\left(\frac{1}{\cosh^2 x}\right) \\
&= \frac{1}{\cosh^2 x + \sinh^2 x} \\
&= \frac{1}{\cosh 2x} \\
&= \operatorname{sech} 2x
\end{aligned}$$